11th Australasian Natural Convection Workshop (11ANCW)
Sydney, Australia, 9-10 December 2019

Analysis of horizontal natural convection under a novel non-Boussinesq approximation

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1. ABSTRACT

Traditionally, the Boussinesq approximation [1] is adopted for numerical simulation of natural convection problems where density variations are ignored except through the gravity term of the momentum equation. This is based on the assumption that the density variations are small, confining their effect to the buoyancy term. In cases where the density variations are significant, the classical Boussinesq approximation may produce inaccurate results. In these cases, non-Boussinesq approach need to be used.

2. NUMERICAL APPROACH

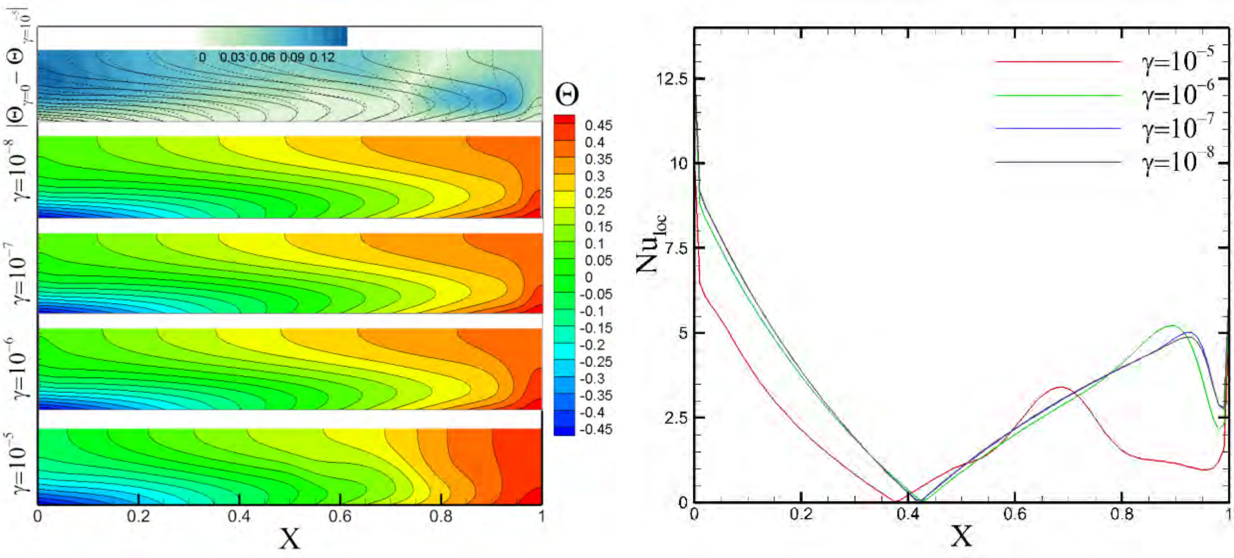
In this study, a new formulation is used [2] in which the density variations are considered in the advection terms in addition to the buoyancy term as follows:

$$\begin{cases} \nabla \cdot \boldsymbol{U} = 0, \\ \partial \boldsymbol{U}/\partial t + (\boldsymbol{U} \cdot \nabla)\boldsymbol{U} = -\nabla P + Pr\,\nabla^2 \boldsymbol{U} - RaPr\Theta\big(\,\boldsymbol{e_g} - \gamma(\boldsymbol{U} \cdot \nabla)\boldsymbol{U}\big), \\ \partial \Theta/\partial t + (\boldsymbol{U} \cdot \nabla)\Theta = \nabla^2 \Theta \end{cases}$$

In the above equations Pr is the Prandtl number and Ra is the Rayleigh number. The non-Boussinesq parameter is defined as $\gamma = \alpha^2/g\,L^3$. The momentum equation is consistent with the momentum equation under the Boussinesq approximation, except for an additional inertial buoyancy term on the right-hand side of the momentum equation. The additional term modifies the effective direction (and strength) of the gravity locally throughout the flow which is ignored in the Boussinesq approximation. Indeed, regions which experience higher spatial accelerations (described by $(\boldsymbol{U} \cdot \nabla)\boldsymbol{U}$) will have significant deviations from the Boussinesq buoyancy approximation. The strength of these deviations relative to the gravity is described by the γ parameter, with $\gamma \to 0$ recovering the classical Boussinesq approximation.

3. RESULTS

The results of the proposed formulation are compared against the Boussinesq approximation for a horizontal natural convection (HNC) problem in a cavity with a height to length ratio H/L = 0.16 in term of local Nusselt number with Rayleigh numbers up to 10^8 (Ra=10^8). Temperature fields at Ra=10^7 corresponding to the different non-Boussinesq parameters (introduced as γ in presented formula) are shown in Fig. 1a as well as the mismatch between temperature fields under the proposed and standard Boussinesq approximations. Fig. 1b shows local Nusselt number along the bottom wall corresponding to the different non-Boussinesq parameters.



(a) Temperature fields at $10^{-5} \leq \gamma \leq 10^{-8}$ (b) Local Nusselt number along the bottom wall

Figure 1. Results at Ra=10^7 (a) Temperature fields corresponding to $10^{-5} \leq \gamma \leq 10^{-8}$. The top frame shows mismatch between temperature fields under the proposed approximation with $\gamma = 10^{-5}$ and the Boussinesq approximation (b) Local Nusselt number along the bottom wall at mentioned γ.

At Ra=10^7, the highest γ value for a steady state solution was found to be 10^{-5} ($\gamma = 10^{-5}$). For $Ra \geq 10^7$ a linear behaviour of the local Nusselt number is observed along the cooling section at 0<X<0.4. For the non-Boussinesq solution with $\gamma = 10^{-5}$, the local Nusselt number has a lower value compared to the lower γ values. A lower and irregular distribution of the local Nusselt number is observed for $\gamma = 10^{-5}$ at 0.7<X<1 which is consistence with a thicker thermal boundary layer in Fig. 1a. Another interesting feature of the local Nusselt number at Ra=10^7 is observed for $\gamma = 10^{-6}$ at 0.8<X<1 where the local Nusselt number has a higher value at 0.8<X<0.9 and a lower value at 0.9<X<1. There is no noticeable difference in terms of the local Nusselt numbers for $\gamma \leq 10^{-7}$.

4. CONCLUSIONS

HNC is studied via a non-Boussinesq approximation in a cavity with a height to length ratio of 0.16. A sample of temperature fields and local Nusselt number at Ra=10^7 for different non-Boussinesq parameters are presented. A significant mismatch between thermo-fluid fields under the proposed and Boussinesq approximations is observed especially for convection-dominated flows which justifies application of the proposed approximation for improved accuracy in natural convection problems at high Rayleigh numbers.

ACKNOWLEDGMENTS

This research was supported by the Australian Research Council through Discovery Projects DP150102920 and DP180102647. The authors are supported by a time allocation on the National Computational Infrastructure (NCI) peak facility through an NCMAS grant. NCI is supported by the Australian Government.

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