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A QUASI-TWO-DIMENSIONAL INVESTIGATION OF UNSTEADY TRANSITION IN SHALLOW FLOW PAST A CIRCULAR CYLINDER IN A CHANNEL

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ABSTRACT

For shallow flow past an obstacle in a channel, the channel depth and blockage ratio play a significant role in the dynamics of the wake. In this study, the flow past a confined circular cylinder is investigated numerically using a spectral element algorithm. The incompressible Navier-Stokes equations are solved over a two dimensional domain, and a linear friction term is added to model the shallow flow around a circular cylinder in a channel: this is known as a quasi-two-dimensional model. A parametric study is performed for the two-dimensional flow by varying the Reynolds number (*Re*) and blockage ratio (β), over the ranges $20 \leq Re \leq 2000$ and $0.2 \leq \beta \leq 0.6$. Subsequently, variation in channel depth is considered by varying the linear friction coefficient, and the resulting time-dependent motion is examined to determine the transition of the flow from steady to unsteady flow as a function of blockage ratio and the linear damping factor. In addition, the wall effects on the Strouhal number and drag coefficients are also investigated. This work lays the foundation for an investigation of magnetohydrodynamic in quasi-two-dimensional channel flows.

NOMENCLATURE

C_D	Total drag coefficient
C_L	Lift coefficient
d	Diameter of the cylinder
f	Frequency of oscillation
F_D	Total drag force per unit length of the cylinder
H	Width of the channel
h	Fluid layer thickness
k	Parameter depending on the velocity profile
Re	Reynolds number, $d\,u/\nu$
Re_c	Critical Reynolds number
St	Strouhal number
$U_{\max}$	Maximum velocity in the channel
U	Average velocity

Greek symbols

β	Blockage ratio, d/H
λ	Linear friction damping coefficient
N	Kinematic viscosity

Subscripts

0	Two-dimensional flow
1,2	Quasi-two-dimensional flow, Q2D

INTRODUCTION

A quasi-two-dimensional (Q2D) flow is one in which a velocity field is dominated by flow in only two of three dimensions. The properties of Q2D flow are relevant in many field of fluid dynamics including nuclear fusion reactors, metallurgy, and crystal growth. In this paper, the shallow wake of the flow behind a confined circular cylinder in a plane duct is analysed using a Q2D model where the vertical velocity is small compared to the horizontal velocity components. In this case the system of Navier-Stokes equations is integrated with respect to the vertical length scale. The behaviour of Q2D flows have studied experimentally by Cardoso *et al.* (1994); Danilov *et al.* (2002); Dolzhanskii *et al.* (1992); Marteau *et al.* (1995); Paret & Tabeling (1997); Tabeling *et al.* (1991). In these experiments, it is assumed that the two-dimensional (2D) approximation for such flows holds, and the friction due to the bottom of the shallow channel can be accounted for with the Rayleigh friction model. The validity of these assumptions have been investigated experimentally by Paret *et al.* (1997) and numerically by Jüttner *et al.* (1997); Satijn *et al.* (2001). The flow past a bluff body in confined domains such as a channel can be significantly influenced by wall effects Mehmet & Owens (2004). The effects of the wall near the circular cylinder at high Reynolds number have been investigated experimentally by Coutanceau & Bouard (1977); Taneda (1965); Gerrard (1978). The results have clearly revealed that the presence of the wall significantly affects the drag force and the vortex shedding from the cylinder. The critical Reynolds number at which the vortex shedding occurs was found to increase with increasing the blockage ratio β.

Chen *et al.* (1995) have performed numerical experiments to investigate the bifurcation for the flow past a cylinder between parallel plates. They focused on the mechanism in which the steady flow past a cylinder at small
Reynolds number loses stability as the Reynolds number is increased. Those numerical experiments were carried out using cylinders with diameters 0.2, 0.5 and 0.7. The flow was perturbed slightly by rotation of the circular cylinder for a short period of time. The resulting time-dependent motions were examined to determine the critical Reynolds numbers beyond which perturbations were amplified, leading to unsteady flow. The numerical results have revealed that steady flow past a confined cylinder loses stability with increasing *Re* through a symmetry-braking Hopf- bifurcation with the value of critical *Re* (based on cylinder diameter) at bifurcation dependent on the blockage ratio. The critical *Re* for $\beta = 0.2$ was found to be 69.

Behr *et al.* (1995) have investigated numerically the influence of the location of the walls on two-dimensional unsteady incompressible flow past a circular cylinder. The case of $Re = 100$ was used as a benchmark. The computations were performed using a space-time velocity-pressure formulation and a velocity-pressure-stress formulation. The results have indicated that the distance between the cylinder and the lateral boundaries have a significant effect on *St*, C_D, and C_L. The minimum distance at which this influence vanishes is approximately 8*d*. Anagnostopoulos *et al.* (1996) have found that the size of the wake vortices decreases with increasing blockage ratio, whereas the spacing between vortices decreases in both directions with increasing blockage ratio when the flow is unsteady. In addition, they found that for a fixed Reynolds number, hydrodynamic forces and Strouhal number (*St*) increase as blockage ratio increases.

Zovatto & Pedrizzetti (2001) have numerically studied the flow past circular cylinder confined in rectangular plane, at various distances from the walls and a range of Reynolds numbers between 100 and 1000. Their results revealed that the critical *Re* at which the transition from steady flow to the periodic vortex shedding regime was found to increase as the cylinder approaches one wall. Furthermore, at high *Re*, the unsteady vortex regime changes from a vortex street to a single row of the same sign vortices as the body approaches one wall. However, for a fixed *Re*, C_D decreased as the cylinder approaches one wall. The results of Cliffe & Tavener (2004) have revealed that for $\beta < 0.6$, the steady symmetric flow loses stability at a critical *Re* less than 300, and remains stable with respect to the time dependent disturbances. However, for β between 0.6 and 0.85, the steady flow is restabilized for $Re < 300$. For $\beta > 0.85$, the first instability changed from time dependent to a steady symmetry-breaking instability. The steady symmetric flow at $\beta = 0.9$ was found to be loss stability to a steady asymmetric flow at $Re = 114.5$.

The effects of the blockage ratio on stability, hydrodynamic forces, and wake structure behind the cylinder for a wide range of blockage ratios ($0.1 \leq \beta \leq 0.4$) and Reynolds number ($Re \leq 280$) have been investigated numerically by Mehmet & Owens (2004). The results have shown that for $Re \leq 20$ and $\beta \leq 0.9$, there were three separate curves of neutral stability: a Hopf-bifurcation to a symmetric state, a pitch-fork bifurcation of a symmetric state to one of two asymmetric states, and a Hopf-bifurcation of an asymmetric state which leads to asymmetric oscillations thereafter. As blockage ratio increased, transition from symmetric vortex shedding to asymmetric vortex shedding occurred. Further increases in the blockage ratios lead to restabilization the flow to a steady asymmetric solution.

Chakraborty *et al.* (2004) have extensively numerically studied the effects of the walls on the steady, two-dimensional flow past a circular cylinder placed symmetrically in a rectangular channel for different ranges of blockage ratio $1.54 \leq \beta \leq 20$ and $0.1 \leq Re \leq 200$. The results have revealed that for a fixed value of *Re*, the total drag was found to increase with increasing blockage ratio, and for a fixed blockage ratio it was decreased with increasing Re. In addition, the length of recirculation, the angle of separation, and the critical Reynolds number at which vortex shedding occurs were found to decrease with increasing *Re* for a fixed value of β.

Unsteady numerical computations have been carried out by Mettu *et al.* (2006) for the flow around an asymmetrically confined circular cylinder in a plane channel. The range of *Re* was between 10 and 50, blockage ratio between 0.1 and 0.4, and the gap ratio between 0.125 and 1. The results demonstrated that the critical *Re* at which the transition occurs from a steady to unsteady flow increases with decreasing the gap ratio for all values of β; however, C_D and *St* were found to increase with decreasing the gap ratio for a fixed *Re*. An oscillation in C_L increases in the negative direction as the gap ratio decreases for a fixed value of β, while it was completely suppressed when the cylinder approaches one wall.

MODEL DESCRIPTION

Quasi 2D Equations

The problem under consideration is unsteady, quasi-two-dimensional and a viscous flow of an incompressible fluid past a circular cylinder confined in a plane channel, as shown schematically in Fig.1. It consists of two plane walls separated by a distance *H*, confined cylinder of diameter *d*. The cylinder is placed at the centre of the duct and located at 12*d* and 42*d* from the inlet and outlet, respectively, following Tezduyar & Shih (1991). The cylinder diameter is $d = 0.01$ m and the blockage ratio $\beta = d/H$ is varied between $\beta = 0.2$-0.6. The vertical velocity component is small compared to the horizontal velocity component thus the vertical dependence can be parameterized by adding an external friction term (Dolzhanskii *et al.* 1992). The Q2D Navier-Stokes equations can be written as

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} = \frac{1}{Re} \nabla^2 \mathbf{u} - \nabla p - \lambda \mathbf{u} \qquad (1)$$

$$\nabla \cdot \mathbf{u} = 0 \qquad (2)$$

where $\mathbf{u}$, *p*, and ν are the velocity vector, kinematic static pressure, and kinematic viscosity, respectively. λ is the friction coefficient due to the effect of the bottom which is equal to $\lambda = k\left(\frac{2\nu}{h^2}\right)$, where h is the fluid layer thickness, k is a parameter depending on the actual velocity profile.

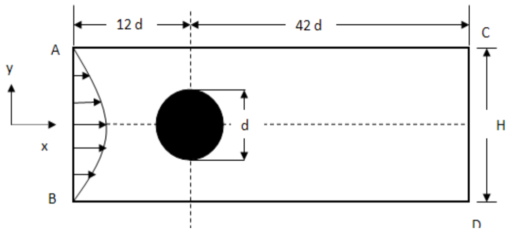

Figure 1: Schematic diagram of the computational domain.

Numerical Methodology

The flow equations (1) and (2) are solved using a spectral-element package (Sheard, Leweke, Thompson & Hourigan 2007; Sheard, Fitzgerald & Ryan 2009). The package utilizes a nodal spectral element method to discretise the flow field in a two-dimensional plane where the flow variables (**u**, *p*) are computed at Gauss-Legender-Lobatto quadrature points. It was found in the computations that great care was required in constructing the mesh in order to obtain accurate results. For example, different kinds of mesh structure were needed around the cylinder diameter d = 0.2 than were needed with cylinder diameter d = 0.6. The degree of polynomial and the number of macro elements for β = 0.2 was 7 and 2108 respectively. The mesh structure for a blockage ratio β = 0.2 is shown in Fig. 2, where the degree of polynomial and the number of the macro elements were 8 and 1609 respectively.

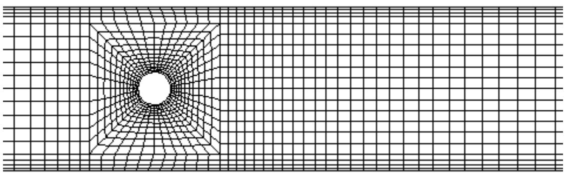

Figure 2: Computational spectral element mesh for β = 0.2.

Zero-valued Dirichlet conditions are imposed on the velocity field on the side walls and cylinder surface. A parabolic velocity profile is imposed through a Dirichlet condition, taking the form

$$u = U_{max}\left[1-\left(\frac{y}{0.5H}\right)\right]^2,$$

where the spatially averaged velocity in the channel is two-thirds of the maximum, $U = 2/3\, U_{\max}$. A constant reference pressure is imposed at the outlet, and a high-order Neumann condition is imposed on Dirichlet velocity boundaries to preserve the third-order time-accuracy of the scheme (Karniadakis, Israeli & Orszag 1991).

The frequency of shedding from the cylinder is normalised as a Strouhal number, which is defined as

$$St = \frac{f\,d}{U}$$

A drag coefficient (two-dimensional, per unit depth) is defined as

$$C_D = \frac{F_D}{0.5\,U^2 d}$$

RESULTS

Numerical simulations have been performed for the range of blockage ratio between 0.2 and 0.6 at various values of the linear friction coefficient, λ. The first case is denoted as λ_0, which corresponds to normal hydrodynamic two-dimensional flow (λ = 0). Two other cases (denoted as λ_1 and λ_2) correspond to Q2D flow for ranges of λ of $\lambda_1 = 25.6/Re$ and $\lambda_2 = 128/Re$, respectively. The present results have been validated by comparing with published data for the case of two-dimensional flows (λ_0). The validation test consists of finding the critical *Re* and *St* corresponding to the transition to the unsteady flow regime. The present computed critical *Re* and *St* numbers are compared with the published results of Chen *et al.* (1995); Mettu *et al.* (2006), as shown in Fig. 3 and Fig. 4, respectively, which demonstrate a good agreement.

A non-linear Landau model (Sheard, Thompson & Hourigan 2004) is implemented in combination with Q2D simulation in order to determine the critical Re for the transition to the unsteady flow regime. The estimation of Re_c for β = 0.2 at λ_0 is shown in Fig. 5. Similarly linear trends were obtained for λ_1 and λ_2. For λ_0, the calculated growth rates revealed that the onset of unsteady flow occurred at approximately $Re_c = 69.2$. For λ_1 and λ_2, this transition occurs at $Re_c = 306$ and 748, respectively. The variation of critical *Re* with λ for the ranges of β between 0.2 and 0.6 is shown in Fig. 6. For a given value of β, Re_c increases significantly with increasing λ, which implies that the transition to the unsteady flow regime is delayed as the linear friction coefficient increases (hence the channel depth decreases). This is due to the resistive action of the linear damping term, which impedes the development of oscillations in the flow. However, for a fixed λ, the critical *Re* decreases as blockage ratio increases for the both cases (λ_1, λ_2) of Q2D flow, while it increases for the case of two dimensional flows (λ_0).

Fig. 7 illustrates the effect of increase β on *St* at different values of λ. For a fixed value of β, (except at 0.6 for the case of λ_2), *St* increases with increasing β. The change in *St* as λ increases from λ_1 to λ_2 is small for the range of β between 0.2 and 0.4, whereas it increases significantly as λ changes from λ_0 to λ_1, almost doubling.

However, the change in *St* as λ increases from λ_0 to λ_1 and λ_2 is negligible as β increases from 0.4 to 0.55. Moreover, its value become smaller than that of λ_0 and λ_1 at β = 0.6. For the case of λ_2, *St* decreases as β increases from 0.2 to 0.4, and then it increases slightly for the rest of range of β. However, *St* increases significantly for both cases of λ_0 and λ_1 as β increases from 0.2 to 0.6.

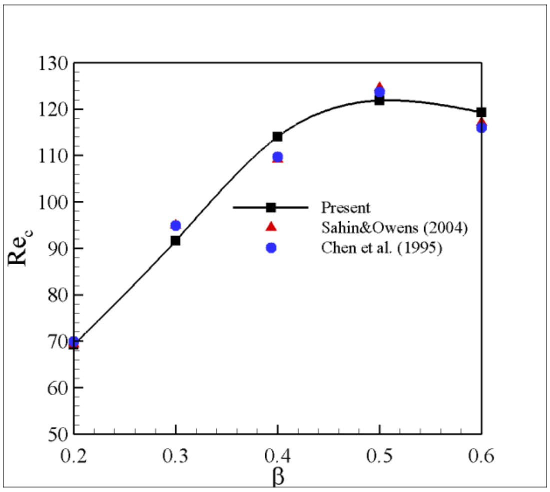

Figure 3: Comparison of Re_c values at various β with the published data for the case of two-dimensional flows

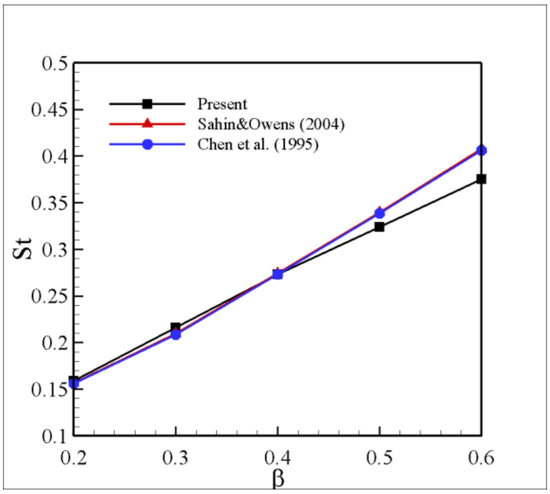

Figure 4: Comparison of *St* at various β against published data for the case two-dimensional flows.

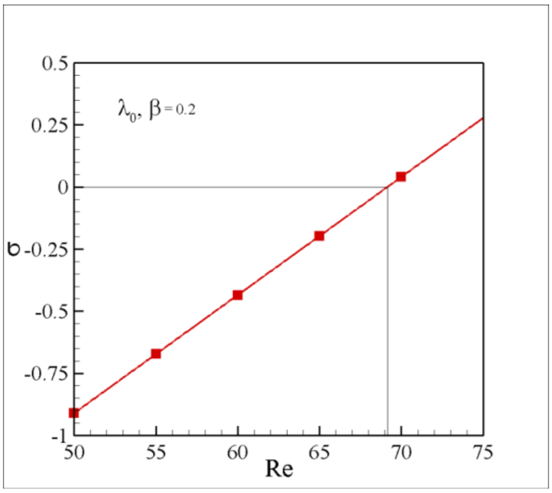

Figure 5: Predicted growth rate as a function of *Re* for the case of two dimensional flows.

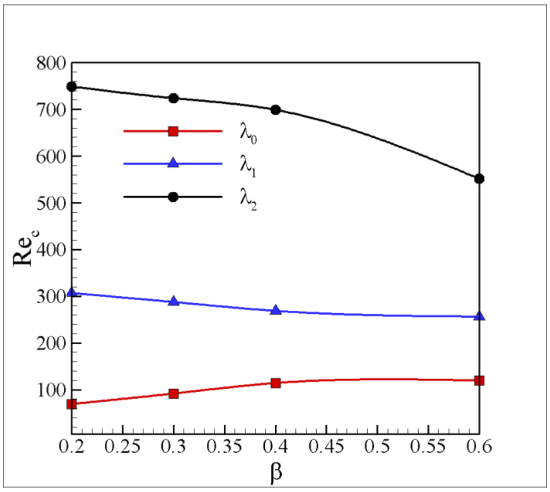

Figure 6: Variation of critical *Re* with β and λ.

The effect of λ on C_D for different values of β is shown in Fig. 8. For the case of λ_0, the values of C_D increase gradually as β increases from 0.2 to 0.4. However, it starts to increase significantly for the rest of β. This is attributed to the fact that as the β becomes bigger, the resistance to the flow increases. As a result, the velocity gradients become sharper which results higher drag force on the cylinder. However, for the case of λ_2, C_D decreases slightly for the range of β between 0.2 and 0.4, and then it starts to increase smoothly as β increases from 0.4 to 0.6. This is attributed to the effect of linear damping friction and the confinement which shifts the transition to higher critical *Re*; therefore the viscous component of the total drag force vanishes and the pressure component increases. Interestingly, at β = 0.2, C_D increases as λ changes from λ_1 to λ_2. However, it begins to increases as β changes from 0.4 to 0.6.

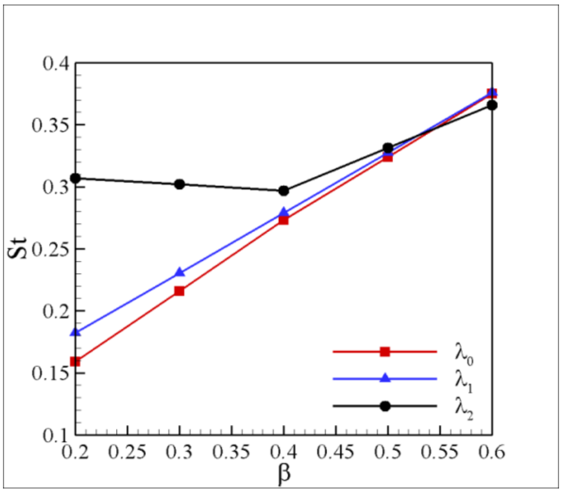

Figure 7: Effects of λ on *St* for different values of β.

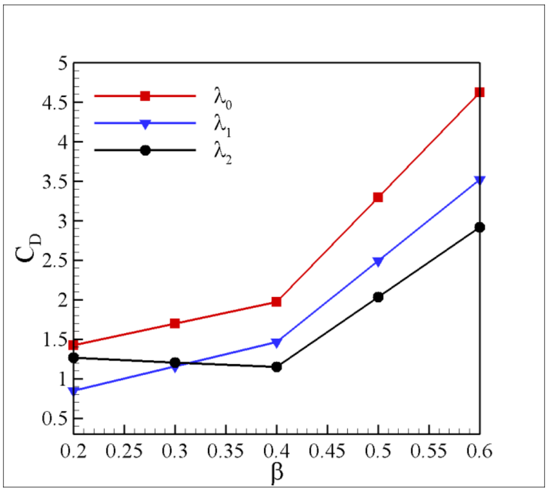

Figure 8: Effects of λ on C_D for different values of β.

In order to elucidate the effect of the linear damping friction coefficient on the flow, the vorticity contours over a single shedding cycle of the relatively mature flow computed at (β = 0.2, λ = 0, Re = 75) and at (β = 0.2, $\lambda = \lambda_1$, Re = 760) are shown in fig. 9 and fig. 10, respectively. These figures indicate the oscillatory nature of the flow of the recirculating flow immediately to the rear of the cylinder. It is clear from the figures that in the case of the Q2D flow, the Karman vortex street is still generated due to the proximity of the wall, which causes the wake vortices entrain vorticity from the walls of the boundary layers into the wake.

CONCLUSION

In this work, unsteady numerical computations have been carried out for quasi-two-dimensional flow past a symmetrically confined circular cylinder in a plane channel for a range of blockage ratio between 0.2 and 0.6, Reynolds numbers up to 5000, and a linear friction damping coefficient, λ, between 0 (2D flow) and 0.323. The transition of the flow around the cylinder is characterized as a function of both β and λ.

The critical *Re* at which the transition occurs from a steady to unsteady flow regime was found to increase significantly as λ increases for a fixed value of β. In contrast to the case of two dimensional flows, the critical *Re* is found to decrease with increasing β for non-zero λ. For a given β, it was found that *St* significantly increases as λ is changed from λ_0 to λ_2 for the range of β between 0.2 and 0.4. However, this effect becomes negligible for β between 0.4 and 0.55. For the range of β between 0.2 and 0.4, as λ increased to λ_2, C_D decreased slightly and then began to increases gradually with further increases in β.

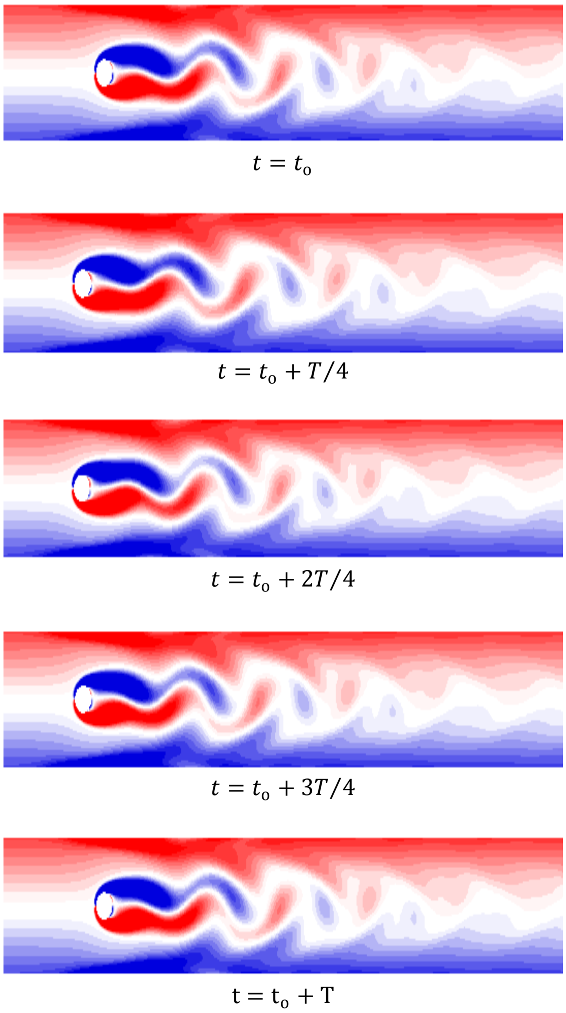

Figure 9: Vorticity contours of the periodic flow at *Re* = 75, β = 0.2 and λ = 0.

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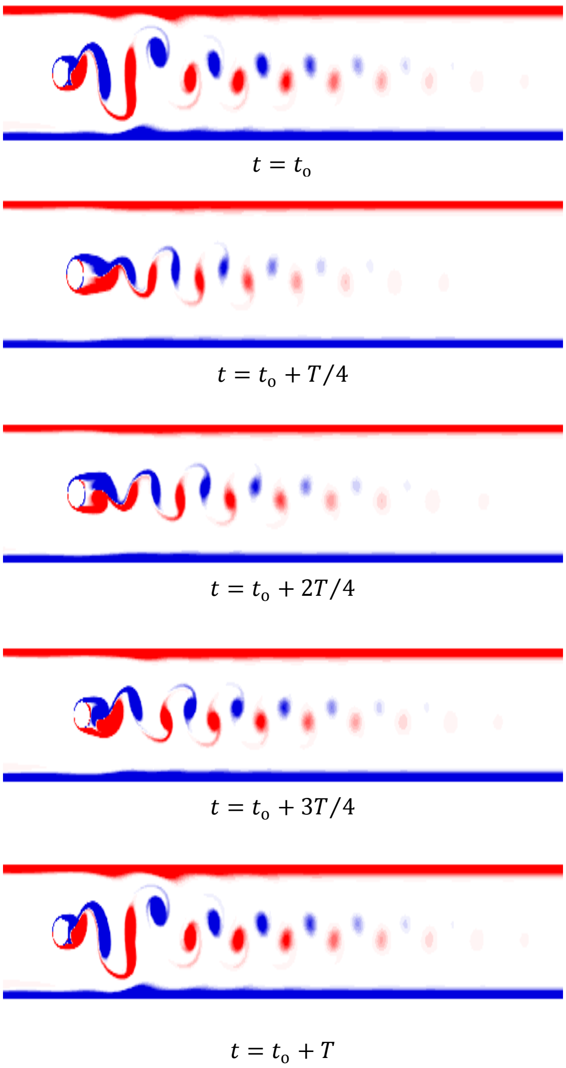

Figure 10: Vorticity contours of the periodic flow at *Re* = 760, β = 0.2 and λ= λ_1

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